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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2016

FIRST YEAR [BATCH 2015-18]

Date : 23/05/2016 **MATHEMATICS FOR ECONOMICS (General)**

Time : 11 am – 2 pm **Paper :** II Full Marks : 75

[Use a separate Answer Book for each group]

Group – A

Answer **any seven** questions of the following : (7 × 5)

1. a) Prove that $\lim_{x\to 0} x\sin\frac{1}{x^2} = 0$ (2)

   b) Evaluate the limit, $\lim_{x\to 0}\frac{e^x - e^{-x} - 2\log(1+x)}{x\sin x}$ (3)

2. a) Define a continuous function $f : D \to \mathbb{R}$. (2)

   b) Define $f : \mathbb{R} \to \mathbb{R}$ by f(x) = 1x1, $x \in \mathbb{R}$ where $\mathbb{R}$ denotes the set of all real numbers. Is f continuous? (3)

3. a) A function $f : [0,1] \to [0,1]$ is continuous on [0,1]. Prove that there exists a point c in [0,1] such than f(c) = c. (2)

   b) A function $f : [0,1] \to \mathbb{R}$ is continuous [0,1] and assumes only rational values. If $f(\frac{1}{2}) = \frac{1}{2}$, prove that $f(x) = \frac{1}{2}$ for all $x \in [0,1]$. (3)

4. a) Let $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function, check whether f is continuous. (3)

   b) Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = |x|, x \in \mathbb{R}$. Is f differentiable on $\mathbb{R}$. (2)

5. a) Let $f : [a,b] \to \mathbb{R}$ be a function such that $f'(x) = 0 \quad \forall x \in [a,b]$. Then prove that f(x) = C for all $x \in [a,b]$, for some real constant C. (3)

   b) Let $f : [a,b] \to \mathbb{R}$ be a function such that $|f(x) - f(y)| < M|x-y|^{\alpha} \ \forall x_1 y \in [a,b]$ for some $M > 0$, $\alpha > 1$. Now prove that f is constant on [a, b]. (2)

6. a) State the Leibnitz's theorem. (2)

   b) If $x + y = 1$, prove that the nth derivative of $x^n y^n$ is $n![y - (n_{C_1})^2 y^{n-1}x + n_{C_2} y^{n-2}x^2 - (n_{C_3})^2 y^{n-3}x^3 ..... + (-1)^n x^n]$. (3)

7. a) State Taylor's theorem on infinite series. (1)

   b) Expand the function f(x) = sinx, in a Taylor Series about x = 0. (4)

8. a) Give an example of a continuous function $f : \mathbb{R} \to \mathbb{R}$, which is not differentiable. (2)

   b) Prove that $\frac{2x}{\pi} < \sin x \quad for\ 0 < x < \frac{\pi}{2}$ (3)

9. Let $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = |x|, x \in \mathbb{R}$. Check whether there exist any maximum or minimum of f. (5)

10. a) Divide the number 10 into two parts such that the sum of their cubes is the least possible. (2)

    b) Decompose the number 36 into two factors such that the sum of their squares is the least possible. (3)

Group – B

Answer **any four** questions of the following : (4 × 10)

11. a) Define a sub space W of a vector space V. (5)

b) Check whether the set $X = \{(a_1, a_2, a_3) \quad 2a_1 - 7a_2 + a_3 = 0\}$ is a subspace of $\mathbb{R}^3$. (5)

12. a) Define linear span of a set of vectors $\{v_1, v_2, ..., v_n\}$ in a vector space V. (5)

b) A linear Transformation T is defined by $T(x, y, z) = (x - y,\ x + 2y,\ y + 3z)$. Show that T is non-singular and determine T^{-1}. (5)

13. a) For what values of 'a' the system of equations is consistent? Solve completely in each consistent case. (6)

$x - y + z = 1$

$x + 2y + 4z = a$

$x + 4y + 6z = a^2$

b) Extend the set S to obtain a basis of the vector space $\mathbb{R}^3$. (4)

$S = \{(1,2,1), (2,1,1)\}$

14. a) Define linear dependence of a set of vectors $\{v_1, v_2, ..., v_n\}$ in a vector space V. Check whether the set of vectors $\{(1,1), (2,3), (4,1)\}$ is linearly independent in $\mathbb{R}^2$. (3 + 2)

b) Determine the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ which maps the basis vectors $(0,1,1), (1,0,1), (1,1,0)$ to the vectors $(2,0,0), (0,2,0), (0,0,2)$. Verify that Nullity T+Rank T=3. (5)

15. a) A Linear Transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ be difined by $T(x_1 x_2, x_3) = (2x_1 + x_2 - x_3, x_2 + 4x_3, x_1 - x_2 + 3x_3)$. Find m(T) relative to the ordered bases {(1,0,0), (0,1,0), (0,0,1)}. Verify Rank T = Rank m(T). (6)

b) Investigate if the subset $\{1+2x+x^2, 3+x^2, x+x^2\}$ of $P_3(\mathbb{R})$ is a basis, where $P_3(\mathbb{R})$ is the vector space of all real polynomials of degree <3. (4)

16. a) $\alpha = \left\{\begin{pmatrix}1&0\\0&0\end{pmatrix}, \begin{pmatrix}0&1\\0&0\end{pmatrix}, \begin{pmatrix}0&0\\1&0\end{pmatrix}, \begin{pmatrix}0&0\\0&1\end{pmatrix}\right\}$,

$\beta = \{1, x, x^2\}$

$T : P_3(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$

be a linear Transformation defined by $T(f(x)) = \begin{pmatrix} f'(0) & 2f(1) \\ 0 & f''(3) \end{pmatrix}$. Find $[T]_\beta^\alpha$. (5)

b) Determine the subspace of $\mathbb{R}^3$ spanned by the vectors $\alpha = (1,2,3),\ \beta = (3,1,0)$. Examine if

i) $(2,1,3)$ is in the subspace.

ii) $(-1,3,6)$ is in the subspace. (5)

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